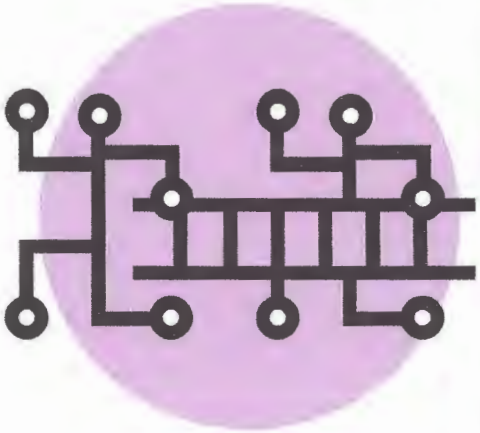
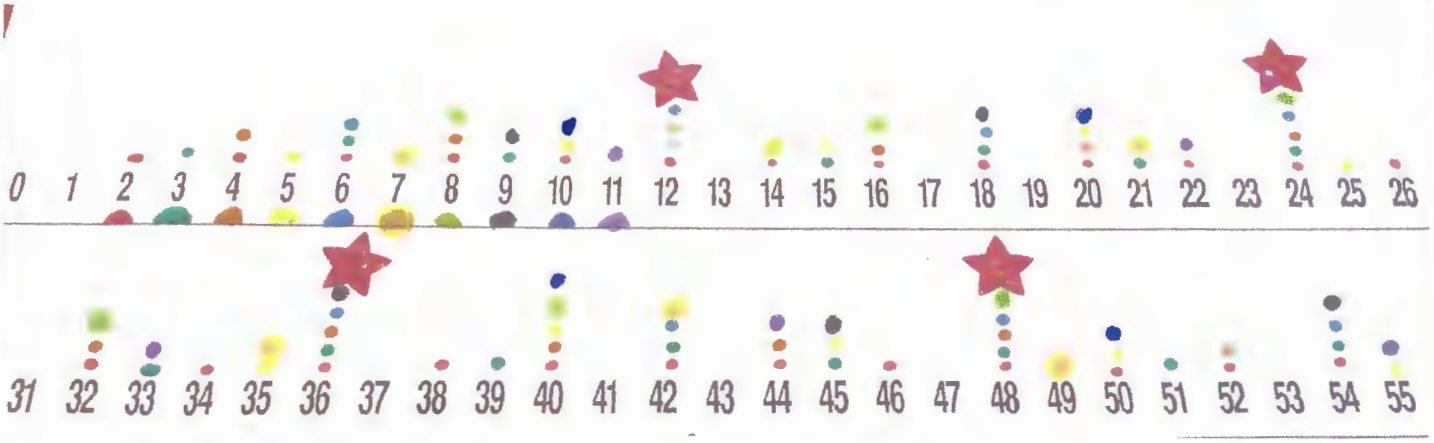
Number Lines and Ladders...

Factoring Strategies that Work!



Activities and Games
for Factoring and Operating
with Fractions.



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Cold Call Questions Regarding Kim's Number Line

Teach Like a Champion-Doug Lemov

Primary Grades (K-2):

- What can you tell me about your class number line?
  (They should articulate it is a growth pattern.)
- What do the colored dots represent?
  (They are part of skip counting patterns. Counting in groups.)
- What number comes before ____ ?
  What number comes after _____ ?
- What multiples of ten is _____ between?
  What is _____ groups of _____ ?
- Pick a number. Ask students what they know about that number?
  (You should hear vocabulary like less than, greater than, skip counting, counting in groups, even/odd, between, before, after, multiples of, coins/bills etc.)

Upper Grades (3-6):

- What can you tell me about your class number line?
  How do you use it as a math tools?
  (They should articulate it is a growth pattern.)
- What do the colored dots represent?
  (They are factors, numbers are multiples.)
- Can you reduce the fraction _____?
  (Have them explain the common factors on number line.)
- Find a number with ___ , ___ , and ___ , as factors.
  How do you know? Tell me the other factors.
- Pick a number. Ask students what they know about that number?
  (You should hear vocabulary like less than, greater than, factors, multiples, even/odd, between, before, after, between what multiples, coins/bills, etc.)

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60

61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90

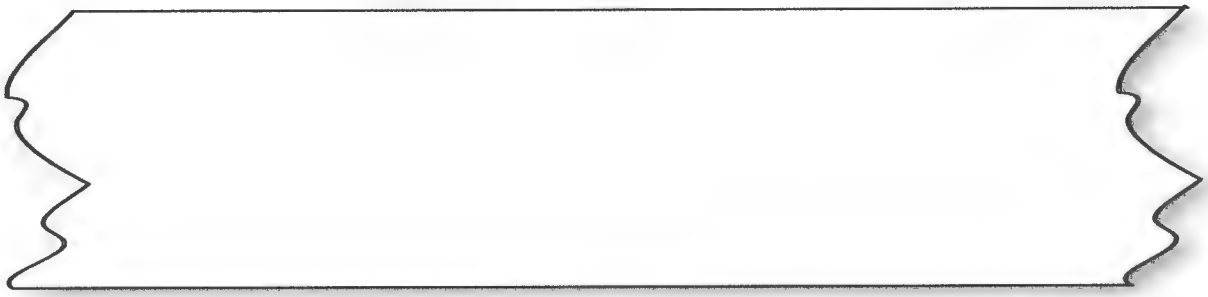
91 92 93 94 95 96 97 98 99 100 101 102 103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119 120

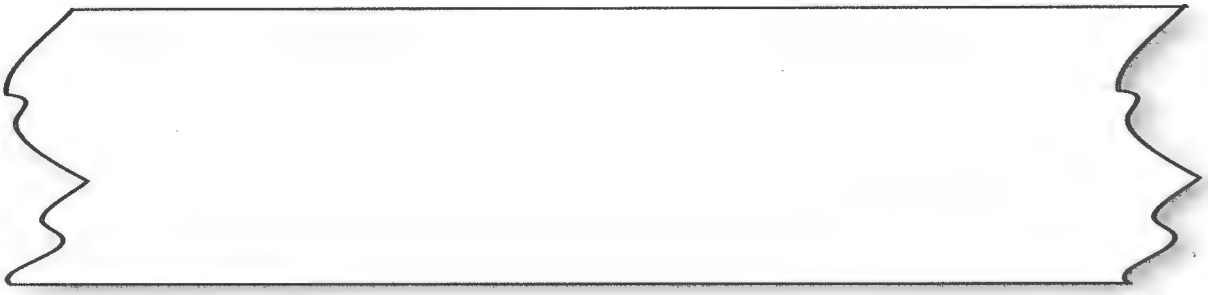
121 122 123 124 125 126 127 128 129 130 131 132 133 134 135 136 137 138 139 140 141 142 143 144

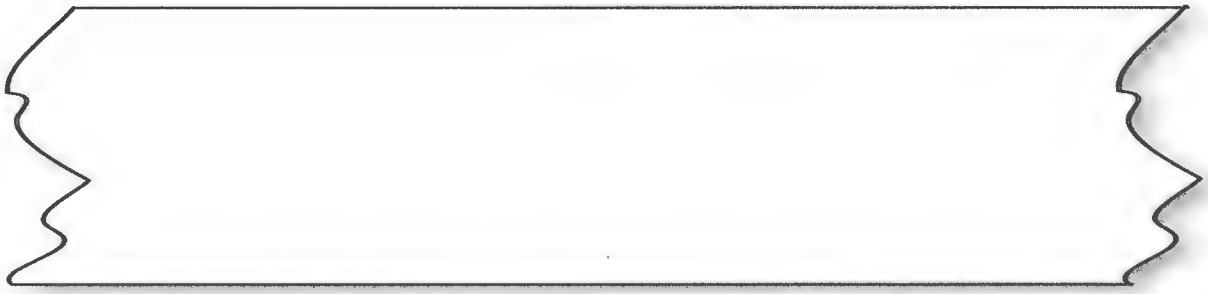
-30 -29 -28 -27 -26 -25 -24 -23 -22 -21 -20 -19 -18 -17 -16 -15 -14 -13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0

-50 -49 -48 -47 -46 -45 -44 -43 -42 -41 -40 -39 -38 -37 -36 -35 -34 -33 -32 -31

Walk The Line







Race Track Fractions

0 $\frac{1}{2}$ $\frac{2}{2}$

0 $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{3}$

0 $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$

0 $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{5}{5}$

0 $\frac{1}{6}$ $\frac{2}{6}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{5}{6}$ $\frac{6}{6}$

Prime Factorization and Factoring
Using the "Ladder" Method

There are several levels to the Ladder.

- Here are all the prime numbers less than 20

2, 3, 5, 7, 11, 13, 17, 19

- Begin factoring with a 2, then a 5, then 3, then 7

The only divisibility rules you need to remember:
*a number is divisible by 2 if it is an even number
*a number is divisible by 5 if it ends in a 0 or a 5
*a number is divisible by 3 if the digits sum to a multiple of 3

Most of the other divisibility rules aren't necessary when using the ladder

Factoring using the Ladder Method

Factoring of one number:

The number to factor is 24

2	24
2	12
2	6
	3

- Ask: will 2 go into 24?.... Yes.
- How many times? (12)
- Will 2 go into 12?..... Yes.
- How many times? (6)
- Will 2 go into 6?.... Yes
- How many times? (3)

So, the Prime Factorization is:
$2 \times 2 \times 2 \times 3$ or $2^3 \times 3$

Notice how nicely and neatly the factors are lined up.
It is more organized than the usual factor tree method.

Another Example:
The number to factor is 52.

2	52
2	26
	13

- Ask: will 2 go into 52?.... Yes.
- How many times? (26)
- Will 2 go into 26?.... Yes.
- How many times? (13)
- Will 2 go into 13?.... No.
- Will 5 go into 13?.... No.
- Recognize that 13 is prime so it cannot be factored any more

So, the Prime Factorization is:

$2 \times 2 \times 13$ or $2^2 \times 13$

How to Factor a numerator and a denominator for simplifying fractions: Use the Ladder to pull out the common prime factors.

You can use the ladder to find the Greatest Common Factor (GCF) and the Least Common Multiple (LCM).

The fraction to simplify is: $\frac{24}{36}$

- Make a "double Ladder" in order to factor out the common factor.
- First Label the numerator and the denominator.

	N	D
2	24	36
2	12	18
3	6	9
	2/ 3	

Ask:
- Will 2 go into 24 and 36?..... Yes.
- How many times for each?
- Will 2 go into 12 and 18?.... Yes.
- How many times for each?
- Will 2 go into 6 and 9?.... No.
- Will 3 go into 6 and 9?.... Yes.
- How many times for each? 3.

In using the ladder, you pull out the common factors leaving the simplified form (numerator and denominator) on the bottom rung of the ladder. So, $\frac{24}{36}=\frac{2}{3}$

As you "grow up" in factoring. You can take out a larger factor if you can recognize it.
For example, on the first rung, if you saw that 6 was a factor of each number you could start with that number. Then they would have to factor 4 and 6 on the next rung.

Finding the GCF and LCM

Using the ladder you can find the Greatest Common Factor GCF and the Least Common Multiple LCM:

	N	D
2	24	36
2	12	18
3	6	9
	2	3

GCF is $2 \times 2 \times 3 = 12$
(Notice how these factors run along the side of the ladder and when multiplied, the result is the greatest common factor.)

LCM is $2 \times 2 \times 3 \times 2 \times 3 = 72$
(Notice how these factors form the shape of an "L" (for Ladder or LCM) and when multiplied, the result is the Least Common Multiple.)

The fractions to add are: $\frac{1}{8}+\frac{2}{6}$

- Make a "double Ladder" in order to factor out the common factors leaving the least common multiple
- Put the denominators into the ladder

2	8	6
	4	3

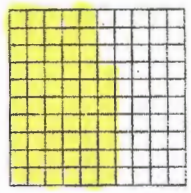
Ask:
- Will 2 go into 8 and 6?..... Yes
- How many times for each?
- Will 2 go into 4 and 3?.... No
- Will 3 go into 4 and 3?.... No
- no more factoring can be done.

Remember, the factors along the outside of the ladder make the Least Common Multiple when multiplied with each other.
So, $2 \times 4 \times 3 = 24$. The LCM is 24
Even better though is how this ladder method sets apart the factors for each denominator.

The bottom rung leaves factors which are not common between the two denominators. Now, the student just has to multiply the two fractions by the missing factor for each denominator.

Example:

$$\frac{1(3)}{8(3)}+\frac{2(4)}{6(4)}=\frac{3}{24}+\frac{8}{24} \quad \text{which is} \quad \frac{11}{24}$$



Wed 11/6

57

Factor

3 | 57 = 30+27 (10+9)

19

$\frac{5}{10} + \frac{7}{100} = \frac{57}{100}$

.5 + .07 = .57

$\frac{57}{100}$ = Already simplified

57%

Factors

1, 3, 19, 57

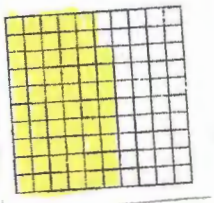
Prime factored

3 · 19

* stomachs in 19 cows.

Thurs, 11-7

58



Factor

2 | 58 (40+18)

29

Factors

1, 2, 29, 58

Prime Factorization

2 · 29

$\frac{5}{10} + \frac{8}{100} = \frac{58}{100}$

.5 + .08 = .58

58%

Simplify:

$\frac{58}{600} = 2\left|\frac{58/100}{29/50}\right. = \frac{29}{50}$

= 2 quarters and 8 pennies

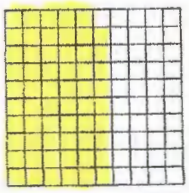
= $1\frac{14}{15}$ of a 30 pak

quarts = 4 × gallons

× 58

232 qts

X



11/8 Fri

59

Factor

59 | 59

1

$\frac{5}{100} + \frac{9}{100} = \frac{59}{100}$

.5 + .09 = .59

59%

Factors

1, 59

Prime

Prime factored

59

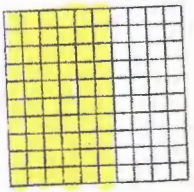
$\frac{59}{100}$ = already simplified because 59 and 100 share no common factors other than 1

W

Consecutive Day

60

11/12/13



2 | 60

2 | 30

3 | 15

5

$\frac{6}{10} = \frac{60}{100}$

.6 = .60

60%

Factors

1, 2, 4, 3, 5, 12, 10, 6, 15, 30, 60,

Prime factors

2 · 2 · 3 · 5

$2^2 \cdot 3 \cdot 5$

$\frac{60}{100} = 20\left|\frac{60/100}{3/5}\right.$

$\frac{63}{5}$ =

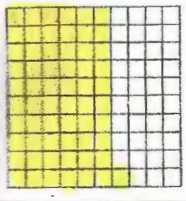
* months in 5 years

* seconds in a minute

* Minutes in an hour.

11/13/13

61



$\frac{6}{10} + \frac{1}{100} = \frac{61}{100}$

.6 + .06 = .61

61%

61 is already simplified

Factor

61 | 61

1

Factors

1, 61

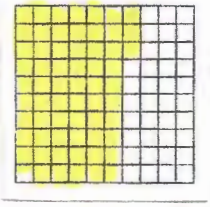
Prime?

Prime factorization

61

11-14-13

62



$\frac{6}{10} + \frac{2}{100} = \frac{62}{100}$

.6 + .02 = .62

62%

Factor

2 | 62

31

Factors:

1, 2, 31, 62

Prime Factorization

2 · 31

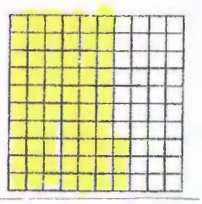
Simplify:

$\frac{62}{100} = 2\left|\frac{62}{100}\right.$

$\frac{31}{50} =$

11/15/13

63



$\frac{6}{10} + \frac{3}{100} = \frac{63}{100}$

.6 + .03 = .63

63%

$\frac{63}{100}$ - already simplified

Factor

3 | 63

3 | 21

7

Factors

1, 3, 7, 9, 21, 63

Prime factored

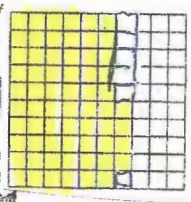
3 · 3 · 7

$3^2 \cdot 7$

- # of stomachs in 21 cows
- 8 hearts in 21 worms
- 6 dimes & 3 pennies

11/19/13

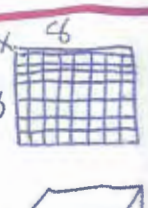
64



$\frac{6}{10} + \frac{4}{100} = \frac{64}{100}$

.6 + .04 = .64

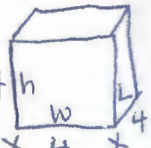
64%



8 by 8

$= 8^2 = 64$

Square #



4 × 4 × 4

64

Cubic Number

Factor

2 | 64

2 | 32

16

1, 2, 4, 8

64, 32, 16

2 | 16

2 | 8

2 | 4

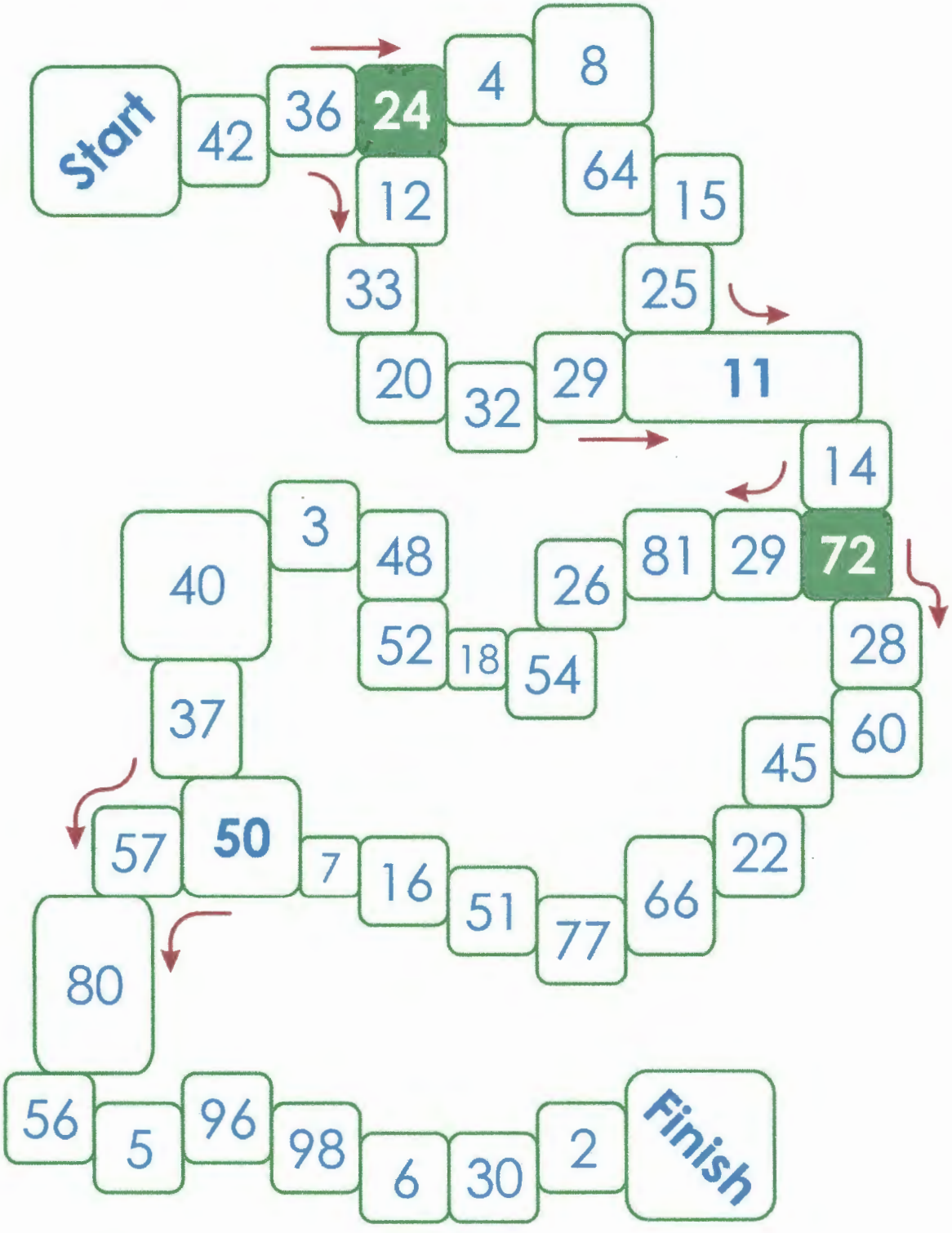
2

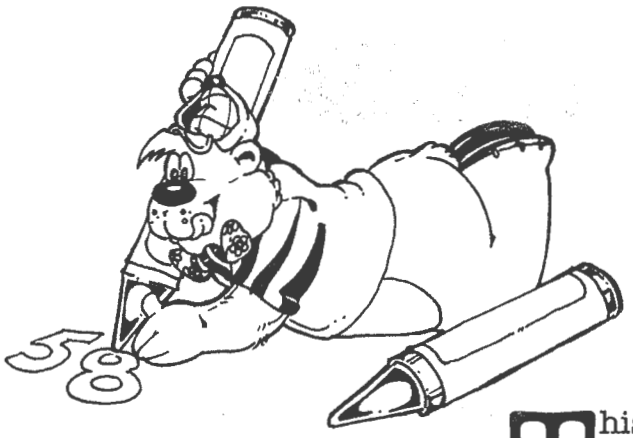
Prime factorization

2 · 2 · 2 · 2 · 2 · 2

2^6

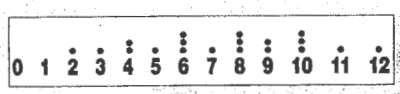
Factor Trail





Factor Fill In Game

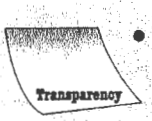
Math Tools:



- Class Number Line

- Overhead Projector

- Overhead Pens



- Transparency of 100 or 144 Worksheet

- Magic Hand

This game is played whole class using the overhead projector and transparencies of either the 100 chart or the 144 chart. The class is divided into two teams. Each team will be represented by a color of the overhead pens.

The first team selects any number on the 100 chart or 144 chart. That number is shaded in by that team's color pen. The other team must state all the factors for the selected number. Those factors would be shaded in by the second team's color. Once a number is shaded in, it cannot be used in the game again.

Play continues back and forth until there are no other numbers to be shaded in. The winning team is the team with the most numbers colored. Players quickly see that if they select a number with few factors, then the other team will get less squares on the board.

For example: Team one is yellow and team two is red. Team one chooses 12 and shades the number on the gameboard. Team two names the factors of 3, 4, 2, 6 and shades the numbers on the gameboard. Team two names the next number and team one names the factors.

				5		7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

	1	2	3	4	5	6	7	8	9	10	11	12
	2	4	6	8	10	12	14	16	18	20	22	24
	3	6	9	12	15	18	21	24	27	30	33	36
	4	8	12	16	20	24	28	32	36	40	44	48
	5	10	15	20	25	30	35	40	45	50	55	60
	6	12	18	24	30	36	42	48	54	60	66	72
	7	14	21	28	35	42	49	56	63	70	77	84
	8	16	24	32	40	48	56	64	72	80	88	96
	9	18	27	36	45	54	63	72	81	90	99	108
	10	20	30	40	50	60	70	80	90	100	110	120
	11	22	33	44	55	66	77	88	99	110	121	132
	12	24	36	48	60	72	84	96	108	120	132	144

Number Line Literature

Algebraic Thinking

Cristoldi, Kathryn. **Even Steven and Odd Todd.** Scholastic, 1996.

Franco, Betsy. **Mathematickles!** McElderry Books, 2003.

Holub, Joan. **Riddle-iculous.** Alber Whitman and Co., 2003.

Hong, Lily Toy. **Two of Everything.** Albert Whitman, 2003. (doubles, groups of)

Hopkins, Lee Bennett. **Marvelous Math.** Simon & Schuster, 1997. (poetry)

Kroll, Virginia. **Equal Shmequal.** Charlesbridge, 2005. (equivalent relationships)

Lee, Cora and O'Reilly, Gillian. **The Great Number Rumble.** Annick Press, 2007.

Lewis, J. Patrick. **Arithmetickle.** Harcourt, 2002.

Losi, Carol. **The 512 Ants on Sullivan Street.** Scholastic, 1997.

Murphy, Stuart. **Earth Day Hooray!** Harper Trophy, 2004. (place value)

Murphy, Stuart. **More or Less.** Harper Trophy, 2005. (comparing numbers)

Neuschwander, Cindy. **Patterns in Peru.** Henry Holt, 2007.

Phillips, Richard. **Numbers-facts, figures and fiction.** Cambridge, 1994.

Walton, Rick. **How Many , How Many, How Many.** Candlewick Press, 1993.

Skip Counting

Hassett, John and Ann. **Cat Up A Tree.** Houghton Mifflin Co., 1998. (fives)

McGrath, Barbara. **The Cheerios Counting Book.** Scholastic, 1998. (tens)

McGrath, Barbara. **The m & m Count to One Hundred Book.** Charlesbridge, 2003.

Murphy, Stuart J. **Leaping Lizards.** Harper Trophy, 2005. (groups of)

Murphy, Stuart. **Spunky Monkeys on Parade.** Harper Trophy, 2001. (2s, 3s, and 4s)

Murphy, Stuart. **Tally O'Malley.** Harper Trophy, 2005.

Pinczes, Elinor. **Arctic Fives Arrive.** Houghton Mifflin, Co., 1996. (fives)

Fractions, Decimals, Per Cents

Adler, David A. **Working with Fractions.** Holiday House, 2007.

Dodds, Dayle Ann. **Full House.** Candlewick Press, 2007.

Leedy, Loreen. **Fraction Action.** Holiday House, 1994.

McGrath, Barbara. **Skittles Riddles Math.** Charlesbridge, 2000.

Murphy, Stuart. **Give Me Half!** Harper Trophy, 2001. (understanding halves)

Murphy, Stuart. **The Grizzly Gazette.** Harper Trophy, 2003. (percentages)

Nagda, Ann Whitehead and Bickel, Cindy. **Polar Bear Math.** Henry Holt, 200.

Napoli, Donna Jo. **The Wishing Club.** Henry Holt, 2007.

Pallotta, Jerry. **The Hershey's Milk Chocolate Fractions Book.** Scholastic, 1999.

Pallotta, Jerry. **Apple Fraction Book.** Scholastic, 2005.

Pallotta, Jerry. **Twizzler Percentages Book.** Scholastic, 2001.

Pilegard, Virginia Walton. **The Warlord's Puppeteers.** Pelican Press, 2003.

Smith, David J. **If the World Were a Village.** Kids Can Press, 2002.

Multiplication and Division

Appelt, Kathi. **Bats on Parade.** Morrow, 1999. (multiplication)

Calvert, Pam. **Multiplying Menace-The Revenge of Rumpelstiltskin.** Charlesbridge, 2005.

Dodds, Dayle Ann. **Minnie's Diner.** Candlewick Press, 2004. (multiplication)

Hutchins, Pat. **The Doorbell Rang.** Mulberry, 1986. (division)

Leedy, Loreen. **2 x 2 = BOO!** Holiday House, 1995. (multiplication)

Nagda, Ann Whitehead. **Cheetah Math.** Henry Holt, 2007.

Neuschwander, Cindy. **Amanda Bean's Amazing Dream.** Scholastic Press, 1998.

Pallotta, Jerry. **The Hershey's Milk Chocolate Multiplication.** Scholastic, 2002.

Pinczes, Elinor. **A Remainder of One.** Houghton Mifflin Co., 1995. (division)

Tang, Greg. **The Best of Times.** Scholastic Press, 2002.

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Here are some of Nancy's favorite resources:

http://xtrememath.com/

Check out this site for student work shown in today's workshop. One of the best things about the eXtreme Math Games, is that the teacher can easily meet the needs of all students. It is easy to differentiate instruction.

http://creativemathematics.com/

Many more number line and factoring activities from Kim Sutton. Check out the Number Line Workbook

http://coolmath.com/

Coolmath.com - An amusement park of math and more... designed for fun! Bored with math? Confused by math? Hate math? Yeah, Coolmath can fix that. Totally dig math? Want to learn more math? Want to get ahead in math? Yep, Coolmath is here for that too! This is a great site for students to use as a resource. I often take students to the computer lab with an assignment which requires them to work through a couple of sessions. You will also find valuable math games.

http://www.mathprojects.com/

The Math Projects Journal is an international publication offering innovative math lessons, discussions of relevant topics, and contributions from teachers from around the world. From 1997-2003, MPJ published its lessons and articles in a hardcopy, bimonthly newsletter. Those lessons have now been compiled in a spiral-bound book, *MPJ's Ultimate Math Lessons*. You will also find many free lesson plans. Chris Shore is the author. Check out the Wallflowers activity for teaching integers.

http://tttpress.com/

Teacher to Teacher Press produces mathematics curriculum written **BY** teachers **TO** teachers and **FOR** students. These are Brad Fulton and Bill Lombard activities. Look for the "The Power of X." Download it by selecting the "Workshops and Conferences" tab and then selecting "Download Conference Materials." The handout is available under the "Conference Materials Archives" section. Email Nancy for a Power Point project for how to do "X" problems.

http://www.boxcarsandoneeyedjacks.com/

This is a great source for a variety of math games. You don't have to reinvent the wheel.

http://mitest.com/

Visit this site for a free online multiple intelligence test for you and/or your students.

http://www.smusd.org//Domain/1211

My school website. Find "Math Menus"; Homework Help Links; Think Like a Mathematician Notes; Common Core Questions and Answers for Parents.

Did you like what you heard and saw today? Let me tailor a Staff Development at your school or district, focusing on concepts that you determine.
I have 10 years of consulting experience at the upper elementary to middle school levels in all areas of mathematics.

Nancy Paulson - Mathematics Instructional Consultant

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